OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE

Unit **4735:** Probability and Statistics 4

Mark Scheme for June 2012

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Annotations

Annotation	Meaning
✓and ✘	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awa.. marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark schem. in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not b. answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the workin. looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect r. must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, awa. the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involve. your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. . not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candida. intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem i. substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an . specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be give. associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation th. of an unknown result.

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Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working follow ... ect for ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would n ... e a candidate passes through the correct answer as part of a wrong argument.

d   When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the s ... otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a p ... dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate ... in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other ... more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e   The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously inco ... Otherwise A and B marks are given for correct work only — differences in notation are of course permitted. A (accurac ... given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage ... may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in th ... rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will ... through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is ... image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-c ...

f   Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates oth ...

Candidates are expected to give numerical answers to an appropriate degree of accuracy. 3 significant figures may of ... this, but this always needs to be considered in the context of the problem in hand. For example, in quoting probabilitie ... we generally expect *some* evidence of interpolation and so quotation to 4 decimal places will often be appropriate. Bu ... always apply – quotations of the standard critical points for significance tests such as 1.96, 1.645, 2.576 (maybe even ... will commonly suffice, especially if the calculated value of a test statistic is nowhere near any of these values. Sensibl ... exercised in such cases.

Discretion must also be exercised in the case of small variations in the degree of accuracy to which an answer is given ... significant figures are expected (either because of an explicit instruction or because the general context of a problem d ... are given, loss of an accuracy ("A") mark is likely to be appropriate; but if 4 significant figures are given, this should no ... penalised. Likewise, answers which are slightly deviant from what is expected in a very minor manner (for example a ... given, after an attempt at interpolation, as 0.6418 whereas 0.6417 was expected) should not be penalised. However, a ... *grossly* over- or under-specified should normally result in the loss of a mark. This includes cases such as, for example ... value of a test statistic is (say) 2.128888446667 merely because that is the value that happened to come off the candi ... that this applies to answers that are given as final stages of calculations; intermediate working should usually be carrie ... a greater degree of accuracy to avoid the danger of premature approximation.

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The situation regarding any particular cases where the accuracy of the answer may be a marking issue ~~~~ deta scheme rationale. If in doubt, contact your Team Leader.

g   Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then e~ the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h   Genuine misreading (of numbers or symbols, occasionally even of text) occurs. If this results in the object and/or diffic being considerably changed, it is likely that all the marks for that question, or section of the question, will be lost. Howe often such that the object and/or difficulty remain substantially unaltered; these cases are considered below.

The simple rule is that *all* method ("M") marks [and of course all independent ("B") marks] remain accessible but at leas ("A") marks do not. It is difficult to legislate in an overall sense beyond this global statement because misreads, even v and/or difficulty remains unchanged, can vary greatly in their effects. For example, a misread of 1.02 as 10.2 (perhaps a sample mean) may well be catastrophic; whereas a misread of 1.6748 as 1.6746 may have so slight an effect as to unnoticeable in the candidate's work.

A misread should normally attract *some* penalty, though this would often be only 1 mark and should rarely if ever be m Commonly in sections of questions where there is a numerical answer either at the end of the section or to be obtained (eg the value of a test statistic), this answer will have an "A" mark that may actually be designated as "cao" [correct ans should be interpreted *strictly* – if the misread has led to failure to obtain this value, then this "A" mark must be withheld marks have been earned. It will also often be the case that such a mark is implicitly "cao" even if not explicitly designa

On the other hand, we commonly allow "fresh starts" within a question or part of question. For example, a follow-throu value of a test statistic is generally allowed (and often explicitly stated as such within the marking scheme), so that the knowledge of how to compare it with a critical value and draw conclusions. Such "fresh starts" are not affected by any

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A misread may be of a symbol rather than a number – for example, an algebraic symbol in a mathematical expression. [...] more likely to bring about a considerable change in the object and/or difficulty of the question; but, if they [...] far as possible in the same way as numerical misreads, *mutatis mutandis*. This also applied to misreads of te [...] cause major problems in fair marking.

The situation regarding any particular cases that arise while you are marking for which you feel you need detailed gu[...] discussed with your Team Leader.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

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Question			Answer	Marks		ce
1		(i)	Successes/variables are indep with same p so B(7+8, p)	B1 **[1]**	or use pgf.	
		(ii)	$P(X = 2\|X + Y = 5) = P(X = 2, Y = 3)/P(X + Y = 5)$	M1		
			$= \binom{7}{2}p^2q^5 \times \binom{8}{3}p^3q^5 / \binom{15}{5}p^5q^{10}$	B1 B1	Numerator Denominator	
			$= 0.392 = (1176/3003) = 56/143$	A1 **[4]**		
2		(i)	$\int_0^\infty 4xe^{-(2-t)x}\,dx$ oe	M1		
			$= \left[\dfrac{-4}{2-t}xe^{-(2-t)x}\right]_0^\infty + \dfrac{4}{2-t}\int_0^\infty e^{-(2-t)x}\,dx$ oe	M1	Using integration by parts (Allow omission of limits for M1M1)	
			$= \dfrac{-4}{(2-t)^2}\left[e^{-(2-t)x}\right]_0^\infty$ oe	A1		
			$= \dfrac{4}{(2-t)^2}$ AG	A1 **[4]**	Allow $\dfrac{4}{(t-2)^2}$	
		(ii)	Requires $E(e^{-Xt}) =$ which is $E(e^{Xt})$ with $-t$ for t	B1 **[1]**	Or from mgfs	or from $\int$ Must be –
		(iii)	$16/(4 - t^2)^2$	B1 **[1]**	AEF, ISW	

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Question			Answer	Marks		nce
3		(i)	Populations have identical/same distributions (apart from location)	B1	Allow 'Data quantitative'	'N
			$(H_0: m_1 = m_2, H_1: m_1 \neq m_2)$			
			Ranks 1 2 4 6 9 10	M1	Can be implied.	
			3 5 7 8 11 12 13			
			$R_m = 32$, $m(m + n + 1) - R_m = 52$	A1		
			$W = 32$	A1	M1A0A1 possible	
			Critical value = 29	B1		
			$32 > 29$, do not reject H	M1	Correct first conclusion ft TS and CV	
			There is insufficient evidence at the 10% significance level of a difference between the median marks of the two groups. oe.	A1 **[7]**	ft TS only.	
		(ii)	Marks should have normal populations with equal variances.	B1 **[1]**	Need 'population'.	NOT popu
		(iii)	2- sample t-test would be better than the Wilcoxon test since it uses more information.	B1 **[1]**	Or is more powerful.	
4		(i)	$E(t^U) = pt + qpt^2 + q^2pt^3 + ...$	M1		
			$= pt(1 + qt + q^2t^2 + ...$	A1	or a=pt, r=qt	or $(1-qt)^-$
			$= pt/1 - qt)$ AG	A1 **[3]**		
		(ii)	For convergence of the infinite series	B1 **[1]**	Or G would be ≤ 0 (or probs or denom)	
		(iii)	$G'(t) = [p(1 - qt) + pqt]/(1 - qt)^2$	M1	or product rule.	
			Mean $= G'(1) = \ldots = 1/p$	M1 A1 **[3]**	CWO	
		(iv)	$G_U = 0.2t/(1 - 0.8t)$; $G_X = [0.2t/(1 - 0.8t)]^4$	B1 B1 **[2]**		
		(v)	Find the coefficient of t^6 in expansion of G_X	M1	Or 3 in the first 5 (B(5, 0.2) and 1 in 6th	=0.0512 x0.2
			$= 0.2^4 \times (4 \times 5/2) \times 0.8^2$	M1		
			$= 0.01024 = 32/3125$	A1 **[3]**		

Question			Answer	Marks		nce
5		(i)	For $n = 4$ $P(X = 0)$ or $P(X = 4) = 2^{-4} = 0.0625$	M1	or $0.9375 < 0.95$	
			$0.0625 > 0.05$ so H_o cannot be rejected	A1		
				[2]		
		(ii)	Sample of times considered random	B1	Allow 'data above or below median'	
			$H_0 : m = 30$, $H_1 : m < 30$	B1	Both hypotheses, median or m	
			Use sign test	M1	May be implied	
			$X \sim B(72, \frac{1}{2})$	M1		No , or wr −1.886 or
			$P(X \le 28) =$ (from N(36,18)	M1		Any other
			Φ $(28.5 \text{ or } 43.5 - 36)/18^{1/2}$	M1	$= (-)1.767$	
			$= 0.0385$ or 0.0386	A1	or $CV = (-)1.645$	0.0297 or
			Or from $B(72, \frac{1}{2}) = 0.0382$	A2	Using calculator procedure	0.038184
			Compare with 0.05 and reject H_0	M1	or $-1.767 < -1.645$	No, or wr
			There is sufficient evidence to accept that the median time for Elena's swims is less than 30s	A1ft	not over-assertive	
				[9]		
6		(i)	Use independence to obtain equation in a and/or b eg $0.4(a+0.08) = 0.08$, $a = (a+b+0.18)(a+0.08)$ $0.18 + 2(b+0.12) + 0.8 = 1.4(0.3 + 2b + 0.4)$	M1	$P(A \cap B) = P(A)P(B)$ or $E(TS) = E(T)E(S)$	$P(S=0)=0$ $P(S=2)=0$ $P(S=0)=0$
			Use independence or $\Sigma p = 1$ or $P(T=1) = 0.6$ to obtain 2nd equation. eg $a + 0.58 + b = 1$ or above	M1		$P(S=2)=0$ $(P(T=1)=$ $a="0.6"x"$
			Correct simplified equation eg $0.4a = 0.048$, $a+b = 0.42$, $0.24 = 0.8b$	A1		M1 $a=0.12$ A
			2nd correct simplified equation	A1		
			$a = 0.12$ AG	A1		
			$b = 0.3$	A1		
				[6]		
		(ii)	$P(T = 2, S = 1) + P(T = 1, S = 0)$	M1		
			$= 0.12 + 0.12 = 0.24$	A1		
				[2]		

Question			Answer	Marks	ce
6		(iii)	$\text{Var}(T) + \text{Var}(S)$	M1	$T - S$: –1 0 1 2)=(
			$\text{Var}(T)= 0.6 + 4 \times 0.4 - (0.6 + 0.8)^2$	M1	p : 0.3 0.38 0.24 0.08
					(M1A1) E(
					used.
			$\text{Var}(S) = 0.3 + 4 \times 0.5 - (0.3 + 1)^2$	M1	$E(T - S) = 0.1$ $E([T-S]^2])$ Allow V
					= 0.86 provided
					E(TS)-E(
					incorrectl
			$\text{Var}(T - S) = 0.85$	A1	Var = 0.86 – 0.01 = 0.85
					(M1A1)
				[4]	
7		(i)	$f(-2) = \frac{1}{4}(1 - 2a) \geq 0 \Rightarrow a \leq \frac{1}{2}$	M1	Using $f(x) \geq 0$
			$f(2) = \frac{1}{4}(1 + 2a) \geq 0 \Rightarrow a \geq -1/2$	A1	Allow omission of ¼
				[2]	
		(ii)	$\int_{-2}^{2} \frac{1}{4}(x + \alpha x^2)\,dx$	M1	
			$= [\,\frac{1}{4}(x^2/2 + ax^3/3)] = 4a/3$	A1	
				[2]	
		(iii)	$E(3X/4) = a \Rightarrow T_1 = 3X_1/4$	M1 A1	
				[2]	
		(iv)	$E(T_2) = \frac{3}{8}(E(X_1) + E(X_2)).$	M1	
			$= \frac{3}{8}(\frac{4}{3}a + \frac{4}{3}a) = a, \Rightarrow T_2$ unbiased for a	A1	
				[2]	
		(v)	$\text{Var}(T_1) = \frac{9}{16}\sigma^2$	M1 A1	M1 for $a^2 \sigma^2$ for either T.
			$\text{Var}(T_2) = \frac{9}{64}(\sigma^2 + \sigma^2) = \frac{9}{32}\sigma^2$	A1	
			$\text{Var}(T_2) < \text{Var}(T_1) \Rightarrow T_2$ better	M1	
				[4]	

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Question			Answer	Marks	nce
8		(i)	$P(A \cap B) = 0.6P(B)$	M1	May be implied.
			$P(A \cap B) \leq P(A) = 0.3$	M1	
			$P(B) \leq 0.3/0.6 = 0.5$ AG	A1	
				[3]	
		(ii)	$P(A \cup B) = x = 0.3 + P(B) - 0.6P(B)$	M1	Use formulae for union and cond prob.
			$P(B) = (x - 0.3)/0.4$	A1	
				[2]	

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Question			Answer	Marks	
8		(i)	$P(A \cap B) = 0.6P(B)$	M1	May be implied.
			$P(A \cap B) \leq P(A) = 0.3$	M1	
			$P(B) \leq 0.3/0.6 = 0.5$ AG	A1	
		(ii)	$P(A \cup B) = x = 0.3 + P(B) - 0.6P(B)$	M1	Use formulae for union and cond prob.
			$P(B) = (x - 0.3)/0.4$	A1	